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THERMO-ELASTIC-PLASTIC ANALYSIS OF A THICK-WALLED CYLINDER WITH TEMPERATURE-DEPENDENT YIELD STRESS

P. C. T. CHEN

FEBRUARY 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER WEAPON SYSTEMS LABORATORY
BENÉT WEAPONS LABORATORY
WATERVLIET N.Y. 12189

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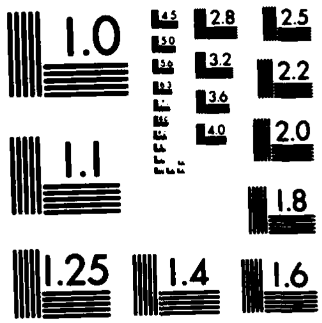

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Numerical Approach
Non-isothermal

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A numerical approach based on the finite-difference method and incremental solution procedure has been developed for analyzing the thermo-elastic-plastic problem of a thick-walled cylinder with temperature-dependent yield stress. The cylinder is subjected to a combination of internal pressure and temperature variation. The material is assumed to obey the von Mises' yield criterion, the associated flow theory, and the isotropic hardening rule. Some numerical results for the displacements and stresses are presented.

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INTRODUCTION

The isothermal elastic-plastic problems of thick-walled cylinders subjected to mechanical and/or thermal loadings have been solved by many investigators based on different theories or methods.[1-4] The yield stress in all isothermal theories is assumed to be temperature-independent. Although good progress has been made recently in developing constitutive relations for thermo-elastic-plastic and time-dependent inelastic theories,[5,6] the research effort in this area has not reached a state of completion. In addition, the general solution of thermo-elastic-plastic problems is still very difficult and frequently very costly.[6-8] As a result, our research has been directed towards the development of a special purpose computer program for solving thick-walled cylinder problems of potential importance to the Army.

This report shows a numerical approach for analyzing the thermo-elastic-plastic problems of thick-walled cylinders with temperature-dependent yield stress. The cylinder is subjected to a combination of internal pressure and temperature variation. The material is assumed to obey the von Mises' yield criterion, the associated flow theory, and the isotropic hardening rule. Some numerical results for the displacements and stresses are presented.

THERMO-ELASTIC-PLASTIC THEORY

For small displacement analysis, the total strain-rate tensor $\dot{\varepsilon}_{ij}$ is composed of corresponding elastic, plastic, and thermal components as follows:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}{}^{e} + \dot{\varepsilon}_{ij}{}^{p} + \dot{\varepsilon}_{ij}{}^{T} \tag{1}$$

References are listed at the end of this report.

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On the basis of the above assumptions, we can readily find that

$$\dot{\varepsilon}_{ij}{}^{P} = \frac{3}{2}\,(\dot{\varepsilon}^{P}/\sigma)s_{ij} \tag{9}$$

and

$$\dot{\lambda} = \dot{\varepsilon}^{P} = [\frac{3}{2}\,\frac{s_{ij}}{\sigma}\,\dot{\sigma}_{ij} - \frac{\partial\sigma}{\partial T}\,\dot{T}]/H' \tag{10}$$

where

$$H' = \frac{\partial\sigma}{\partial\varepsilon^{P}} = \frac{\omega E}{1-\omega}\ , \quad \omega E = \partial\sigma/\partial\varepsilon \tag{11}$$

$$\partial\sigma/\partial T = \partial\sigma_{o}/\partial T + \varepsilon^{P}(\partial H'/\partial T) \tag{12}$$

and σ_o is the initial yield stress.

Substituting Eqs. (2) and (9) into Eq. (1), one obtains the general constitutive equations relating $\dot{\varepsilon}_{ij}$ to $\dot{\sigma}_{ij}$ and $\dot{T}$. For numerical solutions by the finite-element method or the finite-difference method, it is desirable to find the inverse form which relates $\dot{\sigma}_{ij}$ to $\dot{\varepsilon}_{ij}$ and $\dot{T}$. For the isotropic-hardening, thermo-elastic-plastic theory, the explicit inverse relationships are given below in a form slightly different from that shown in Reference 7.

$$\dot{\sigma}_{ij} = \frac{E}{1+\nu}\,[\delta_{ik}\delta_{jl} + \frac{\nu}{1-2\nu}\,\delta_{ij}\delta_{kl} - \frac{1}{s}\,s_{ij}s_{kl}]\dot{\varepsilon}_{kl}$$

$$- [\frac{E}{1-2\nu}\,\alpha\,\delta_{ij} - \frac{(\partial\sigma/\partial T)}{1+H'/3G}\,\frac{s_{ij}}{\sigma}]\dot{T} \tag{13}$$

where

$$s = \frac{2}{3}\,\sigma^{2}(1 + \frac{1}{3}\,\frac{H'}{G}) \tag{14}$$

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EQUATIONS FOR THICK-WALLED CYLINDER

For the isotropic-hardening, thermo-elastic-plastic thick-walled cylinders, the incremental form of Eq. (13) reduces to

$$\Delta\sigma_i = d_{ij}\, \Delta\varepsilon_j - \Delta\sigma_i^{\circ} \quad , \quad i = r,\theta,z \tag{15}$$

where

$$d_{ij} = \frac{E}{1+\nu} \left(\frac{\nu}{1-2\nu} + \delta_{ij} - \frac{1}{s}\, \sigma_i'\sigma_j'\right)$$

$$\Delta\sigma_i^{\circ} = \left[\frac{E\alpha}{1-2\nu} - \frac{(\partial\sigma/\partial T)}{1+H'/3G}\, \frac{\sigma_i'}{\sigma}\right]\Delta T \tag{16}$$

$$\sigma_i' = \sigma_i - \sigma_m \quad , \quad \sigma_m = (\sigma_r+\sigma_\theta+\sigma_z)/3$$

In the quasi-static state with no body forces, the radial and tangential stresses must satisfy the equilibrium equation,

$$r(\partial\sigma r/\partial r) = \sigma_\theta - \sigma_r \tag{17}$$

and the corresponding strains must satisfy the compatibility equation

$$r(\partial\varepsilon_\theta/\partial r) = \varepsilon_r - \varepsilon_\theta \tag{18}$$

Consider a thick-walled cylinder of inner radius a and external radius b. The cylinder is subjected to inner pressure and temperature (p and T_a), external pressure and temperature (q and T_b), and end force (f). The boundary conditions for the generalized plane-strain conditions are

$$\sigma_r(a,t) = -p \quad , \quad T(a,t) = T_a \tag{19}$$

$$\sigma_r(b,t) = -q \quad , \quad T(b,t) = T_b \tag{20}$$

$$2\pi\int_a^b r\sigma_z dr = \mu\pi a^2 p + f \tag{21}$$

where μ is 0 or 1 for open-end or closed-end conditions, respectively. The temperature distribution must satisfy the heat conduction equation subjected to boundary conditions (19) and (20),

The sum of elastic and thermal strain-rates is assumed to be determined by the Duhamel-Neumann law,

$$\dot{\varepsilon}_{ij}{}^{e} + \dot{\varepsilon}_{ij}{}^{T} = E^{-1}[(1+\nu)\ \dot{\sigma}_{ij} - \nu\delta_{ij}\ \dot{\sigma}_{kk}] + \alpha\dot{T}\delta_{ij} \tag{2}$$

in which E is Young's modulus, ν is Poisson's ratio, α is the thermal expansion coefficient, T is the rate of temperature change, δ_{ij} is the Kronecker delta, and σ_{ij} is the stress tensor.

The plastic strain-rate $\dot{\varepsilon}_{ij}{}^{P}$ is derivable from the plastic potential $g(\sigma_{ij})$ by the normality condition

$$\dot{\varepsilon}_{ij}{}^{P} = \dot{\lambda}\ \partial g/\partial\sigma_{ij} \tag{3}$$

where $\dot{\lambda}$ is a positive scalar variable.

The yield function for non-isothermal isotropic strain-hardening material can be written as

$$F = f(\sigma_{ij}) - \sigma\ (\varepsilon^{P},T) \tag{4}$$

where

$$\varepsilon^{P} = \int \dot{\varepsilon}^{P}\ dt \tag{5}$$

$$\dot{\varepsilon}^{P} = (\frac{3}{2}\ \dot{\varepsilon}_{ij}{}^{P}\ \dot{\varepsilon}_{ij}{}^{P})^{1/2} \tag{6}$$

and $\sigma(\varepsilon^{P},T)$ represents the dependence of yield stress on the accumulated increments of effective plastic-strain and temperature. When the von Mises' yield criterion and associated flow rule are adopted,

$$f = g = (\frac{3}{2}\ s_{ij}\ s_{ij})^{1/2} \tag{7}$$

and

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\ \sigma_{kk}\delta_{ij} \tag{8}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{k}\frac{\partial T}{\partial t} \tag{22}$$

where k denotes thermal diffusivity. For the special case of steady state distribution, the temperature is given by

$$T = T_a + (T_b - T_a)\log(r/a)/\log(b/a) \tag{23}$$

INCREMENTAL FINITE-DIFFERENCE FORMULATIONS

For loading beyond the elastic limit, an incremental approach of the finite-difference formulation is used. The cross-section of the tube is divided into n rings with $r_1 = a, r_2, \ldots, r_k = \rho, \ldots, r_{n+1} = b$, where ρ is the radius of the elastic-plastic interface. At the beginning of each increment of loading, the distribution of temperature, displacements, strains, and stresses is assumed to be known and we want to determine Δu, $\Delta\varepsilon_r$, $\Delta\varepsilon_\theta$, $\Delta\varepsilon_z$, $\Delta\sigma_r$, $\Delta\sigma_\theta$, $\Delta\sigma_z$ at all grid points for the applied incremental loading, Δp, Δq, Δf, ΔT_i ($i = 1$ to $n+1$). Since the incremental stresses are related to the incremental strains by the incremental form (Eq. (15)) and $\Delta u = r\Delta\varepsilon_\theta$, there exist only three unknowns at each station that have to be determined for each increment of loading. Accounting for the fact that the axial strain ε_z is independent of r, the unknown variables in the present formulation are $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, for $i = 1,2,\ldots n,n+1$, and $\Delta\varepsilon_z$.

The equation of equilibrium (17) and the equation of compatibility (18) are valid for both the elastic and the plastic regions of a thick-walled tube. The finite-difference forms of these two equations at $i = 1,\ldots,n$ are given by

$$c_1(\Delta\sigma_r)_i + c_2(\Delta\sigma_\theta)_i + c_3(\Delta\sigma_r)_{i+1} = c_5 \tag{24}$$

and

$$c_1(\Delta\varepsilon_\theta)_i + c_2(\Delta\varepsilon_r)_i + c_3(\Delta\varepsilon_\theta)_{i+1} = c_4 \tag{25}$$

On the basis of the above assumptions, we can readily find that

$$\dot{\varepsilon}_{ij}P = \frac{3}{2}(\dot{\varepsilon}P/\sigma)s_{ij} \tag{9}$$

and

$$\dot{\lambda} = \dot{\varepsilon}P = [\frac{3}{2}\frac{s_{ij}}{\sigma}\dot{\sigma}_{ij} - \frac{\partial\sigma}{\partial T}\dot{T}]/H' \tag{10}$$

where

$$H' = \frac{\partial\sigma}{\partial\varepsilon P} = \frac{\omega E}{1-\omega}\ , \quad \omega E = \partial\sigma/\partial\varepsilon \tag{11}$$

$$\partial\sigma/\partial T = \partial\sigma_o/\partial T + \varepsilon P(\partial H'/\partial T) \tag{12}$$

and σ_o is the initial yield stress.

Substituting Eqs. (2) and (9) into Eq. (1), one obtains the general constitutive equations relating $\dot{\varepsilon}_{ij}$ to $\dot{\sigma}_{ij}$ and $\dot{T}$. For numerical solutions by the finite-element method or the finite-difference method, it is desirable to find the inverse form which relates $\dot{\sigma}_{ij}$ to $\dot{\varepsilon}_{ij}$ and $\dot{T}$. For the isotropic-hardening, thermo-elastic-plastic theory, the explicit inverse relationships are given below in a form slightly different from that shown in Reference 7.

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$$- [\frac{E}{1-2\nu}\alpha\,\delta_{ij} - \frac{(\partial\sigma/\partial T)}{1+H'/3G}\frac{s_{ij}}{\sigma}]\dot{T} \tag{13}$$

where

$$s = \frac{2}{3}\sigma^2(1 + \frac{1}{3}\frac{H'}{G}) \tag{14}$$

where

$$c_1 = r_{i+1} - 2r_i \quad , \quad c_2 = -r_{i+1} + r_i \quad , \quad c_3 = r_i$$

$$c_4 = (r_{i+1}-r_i)(\varepsilon_r-\varepsilon_\theta)_i - r_i[(\varepsilon_\theta)_{i+1} - (\varepsilon_\theta)_i]$$

$$c_5 = (r_{i+1}-r_i)(\sigma_\theta-\sigma_r)_i - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \tag{26}$$

Substitution of the incremental stress-strain relations (15) into Eq. (24) leads to

$$c_6(\Delta\varepsilon_\theta)_i + c_7(\Delta\varepsilon_r)_i + c_8(\Delta\varepsilon_\theta)_{i+1} + c_9(\Delta\varepsilon_r)_{i+1} + c_{10}\,\Delta\varepsilon_z = c_{11} \tag{27}$$

where

$$c_6 = c_1(d_{12})_i + c_2(d_{22})_i \quad , \quad c_8 = c_3(d_{12})_{i+1}$$

$$c_7 = c_1(d_{11})_i + c_2(d_{21})_i \quad , \quad c_9 = c_3(d_{11})_{i+1}$$

$$c_{10} = c_1(d_{13})_i + c_2(d_{23})_i + c_3(d_{13})_{i+1}$$

$$c_{11} = c_1(\Delta\sigma_r^*)_i + c_2(\Delta\sigma_\theta^*)_i + c_3(\Delta\sigma_r^*)_{i+1}$$

$$+ c_2(\sigma_r-\sigma_\theta)_i + c_3[(\sigma_r)_i - (\sigma_r)_{i+1}] \tag{28}$$

The finite-difference forms of the boundary conditions (19), (20), and (21) are

$$(d_{12})_1(\Delta\varepsilon_\theta)_1 + (d_{11})_1(\Delta\varepsilon_r)_1 + (d_{13})_1\,\Delta\varepsilon_z = -\Delta p + (\Delta\sigma_r^*)_1 \tag{29}$$

$$(d_{12})_{n+1}(\Delta\varepsilon_\theta)_{n+1} + (d_{11})_{n+1}(\Delta\varepsilon_r)_{n+1} + (d_{13})_{n+1}\,\Delta\varepsilon_z = -\Delta q + (\Delta\sigma_r^*)_{n+1} \tag{30}$$

and

$$\sum_{i=1}^{n} [c_{12}^i(\Delta\varepsilon_\theta)_i + c_{13}^i(\Delta\varepsilon_r)_i + c_{14}^i(\Delta\varepsilon_\theta)_{i+1} + c_{15}^i(\Delta\varepsilon_r)_{i+1}]$$

$$+ \left(\sum_{i=1}^{n} c_{16}^i\right)\Delta\varepsilon_z = \mu a^2\Delta p + \Delta f/\pi + \sum_{i=1}^{n} c_{17}^i \tag{31}$$

EQUATIONS FOR THICK-WALLED CYLINDER

For the isotropic-hardening, thermo-elastic-plastic thick-walled cylinders, the incremental form of Eq. (13) reduces to

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In the quasi-static state with no body forces, the radial and tangential stresses must satisfy the equilibrium equation,

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$$2\pi\int_a^b r\sigma_z dr = \mu\pi a^2 p + f \tag{21}$$

where μ is 0 or 1 for open-end or closed-end conditions, respectively. The temperature distribution must satisfy the heat conduction equation subjected to boundary conditions (19) and (20),

where

$$\Delta r_i = r_{i+1} - r_i \quad , \quad c_{12}{}^i = (\Delta r_i) r_i (d_{23})_i$$

$$c_{13}{}^i = (\Delta r_i) r_i (d_{13})^i \quad , \quad c_{14}{}^i = (\Delta r_{i+1}) r_{i+1} (d_{23})_{i+1}$$

$$c_{15}{}^i = (\Delta r_i) r_{i+1} (d_{13})_i$$

$$c_{16}{}^i = (\Delta r_i)[r_i (d_{33})_i + r_{i+1} (d_{33})_{i+1}]$$

$$c_{17}{}^i = (\Delta r_i)[r_i (\Delta\sigma_z{}^\circ)_i + r_{i+1} (\Delta\sigma_z{}^\circ)_{i+1}] \tag{32}$$

Now we can form a system of 2n+3 equations for solving 2n+3 unknowns, $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, at $i = 1,2,\ldots,n,n+1$ and $\Delta\varepsilon_z$. Equations (29), (30), and (31) are taken as first and last two equations, respectively, and the other 2n equations are set up at $i = 1,2,\ldots,n$ using equations (25) and (27). The final system is an unsymmetric matrix of arrow type with the nonzero terms appearing in the last row and column and others clustered about the main diagonal, two below and one above.

NUMERICAL RESULTS AND DISCUSSIONS

The numerical results for two particular problems follow. The first problem is a closed-end thick-walled cylinder subjected to varying internal pressure p and temperature T as shown in Figure 1. The heating is uniform throughout the thickness, but the initial yield stress is temperature-dependent as shown in Figure 1. The other material constants are E = 86,666 psi, $\nu = 0.3$, $\omega = 0.0$, $\alpha = 0.0$. The numerical results for the radial displacements (u_a and u_b) at the inside and outside surface are shown in Figure 2. The percentage of plastic zone is also shown in the figure by the dotted line. The entire cylinder is elastic during the time interval 10 to 12. The results for the three stress components at selected time t = 4, 8,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{k}\frac{\partial T}{\partial t} \tag{22}$$

where k denotes thermal diffusivity. For the special case of steady state distribution, the temperature is given by

$$T = T_a + (T_b - T_a)\log(r/a)/\log(b/a) \tag{23}$$

INCREMENTAL FINITE-DIFFERENCE FORMULATIONS

For loading beyond the elastic limit, an incremental approach of the finite-difference formulation is used. The cross-section of the tube is divided into n rings with $r_1=a, r_2, \ldots, r_k=\rho, \ldots, r_{n+1}=b$, where ρ is the radius of the elastic-plastic interface. At the beginning of each increment of loading, the distribution of temperature, displacements, strains, and stresses is assumed to be known and we want to determine Δu, $\Delta\varepsilon_r$, $\Delta\varepsilon_\theta$, $\Delta\varepsilon_z$, $\Delta\sigma_r$, $\Delta\sigma_\theta$, $\Delta\sigma_z$ at all grid points for the applied incremental loading, Δp, Δq, Δf, ΔT_i (i = 1 to n+1). Since the incremental stresses are related to the incremental strains by the incremental form (Eq. (15)) and $\Delta u = r\Delta\varepsilon_\theta$, there exist only three unknowns at each station that have to be determined for each increment of loading. Accounting for the fact that the axial strain ε_z is independent of r, the unknown variables in the present formulation are $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, for $i = 1,2,\ldots n,n+1$, and $\Delta\varepsilon_z$.

The equation of equilibrium (17) and the equation of compatibility (18) are valid for both the elastic and the plastic regions of a thick-walled tube. The finite-difference forms of these two equations at $i = 1,\ldots,n$ are given by

$$c_1(\Delta\sigma_r)_i + c_2(\Delta\sigma_\theta)_i + c_3(\Delta\sigma_r)_{i+1} = c_5 \tag{24}$$

and

$$c_1(\Delta\varepsilon_\theta)_i + c_2(\Delta\varepsilon_r)_i + c_3(\Delta\varepsilon_\theta)_{i+1} = c_4 \tag{25}$$

10, and 13 are shown in Figures 3 through 5. The differences for the displacements and stresses at t = 4 and 8 clearly demonstrate the effect of temperature-dependence of the yield stress. The same problem with the plane-strain condition has been solved by the Automatic Dynamic Incremental Analysis (ADINA) program.[8] For comparison purposes, the ADINA results for the radial displacement at the outside and the residual stress distribution through the wall at time $\tau = 10$ are also shown in Figures 2 and 4. The agreement is excellent for the stresses and good for the displacement. The small differences in the displacement response may be due to the end conditions and the methods of approaches. The numerical results reported here are based on the finite-difference formulations with $n = 100$.

As a second example, let us consider a closed-end tube subjected to inner temperature T_a only. The numerical results were based on the following parameters: $b = 2"$, $a = 1"$, $n = 100$, $\nu = 0.3$, $E = 30 \times 10^6$ psi, $\omega = 0.0$, $\alpha = 7.75 \times 10^{-6}$ in./in./°F, $\sigma_0 = 30 \times 10^3$ psi, $\sigma/\sigma_0 = 1.0 - T/2 \times 10^{-3}$/°F. When the temperature gradient is of sufficient magnitude, yielding will first expand from the inside. At larger temperature gradient, the plastic zone will expand from both the inside and outside surface toward the interior. The relation between the inside temperature and elastic-plastic interface is shown in Figure 6. The stresses in a closed-end cylinder subjected to temperature gradient of 400°F are shown in Figure 7. The special case when the yield stress is assumed to be temperature-independent was considered in an earlier paper.[4] Those earlier results are shown in Figures 6 and 7 by the dotted lines. A comparison of the results between the solid and dotted lines shows the effects of temperature-dependence of the yield stress.

where

$$c_1 = r_{i+1} - 2r_i \quad , \quad c_2 = -r_{i+1} + r_i \quad , \quad c_3 = r_i$$

$$c_4 = (r_{i+1}-r_i)(\varepsilon_r-\varepsilon_\theta)_i - r_i[(\varepsilon_\theta)_{i+1} - (\varepsilon_\theta)_i]$$

$$c_5 = (r_{i+1}-r_i)(\sigma_\theta-\sigma_r)_i - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \tag{26}$$

Substitution of the incremental stress-strain relations (15) into Eq. (24) leads to

$$c_6(\Delta\varepsilon_\theta)_i + c_7(\Delta\varepsilon_r)_i + c_8(\Delta\varepsilon_\theta)_{i+1} + c_9(\Delta\varepsilon_r)_{i+1} + c_{10}\,\Delta\varepsilon_z = c_{11} \tag{27}$$

where

$$c_6 = c_1(d_{12})_i + c_2(d_{22})_i \quad , \quad c_8 = c_3(d_{12})_{i+1}$$

$$c_7 = c_1(d_{11})_i + c_2(d_{21})_i \quad , \quad c_9 = c_3(d_{11})_{i+1}$$

$$c_{10} = c_1(d_{13})_i + c_2(d_{23})_i + c_3(d_{13})_{i+1}$$

$$c_{11} = c_1(\Delta\sigma_r^{\circ})_i + c_2(\Delta\sigma_\theta^{\circ})_i + c_3(\Delta\sigma_r^{\circ})_{i+1} + c_2(\sigma_r-\sigma_\theta)_i + c_3[(\sigma_r)_i - (\sigma_r)_{i+1}] \tag{28}$$

The finite-difference forms of the boundary conditions (19), (20), and (21) are

$$(d_{12})_1(\Delta\varepsilon_\theta)_1 + (d_{11})_1(\Delta\varepsilon_r)_1 + (d_{13})_1\,\Delta\varepsilon_z = -\Delta p + (\Delta\sigma_r^{\circ})_1 \tag{29}$$

$$(d_{12})_{n+1}(\Delta\varepsilon_\theta)_{n+1} + (d_{11})_{n+1}(\Delta\varepsilon_r)_{n+1} + (d_{13})_{n+1}\,\Delta\varepsilon_z = -\Delta q + (\Delta\sigma_r^{\circ})_{n+1} \tag{30}$$

and

$$\sum_{i=1}^{n} [c_{12}^{i}(\Delta\varepsilon_\theta)_i + c_{13}^{i}(\Delta\varepsilon_r)_i + c_{14}^{i}(\Delta\varepsilon_\theta)_{i+1} + c_{15}^{i}(\Delta\varepsilon_r)_{i+1}] + (\sum_{i=1}^{n} c_{16}^{i})\Delta\varepsilon_z = \mu a^2\Delta p + \Delta f/\pi + \sum_{i=1}^{n} c_{17}^{i} \tag{31}$$

This report presents the numerical results for a closed-end cylinder subjected to varying internal pressure and/or temperature. The thermal problem is due to uniform heating or thermal gradient. If the temperature distribution is solved by a transit analysis, the corresponding thermal stresses can be calculated. The result of this transit thermal problem is to be reported at a later date.

where

$$\Delta r_i = r_{i+1} - r_i \quad , \quad c_{12}{}^i = (\Delta r_i) r_i (d_{23})_i$$

$$c_{13}{}^i = (\Delta r_i) r_i (d_{13})^i \quad , \quad c_{14}{}^i = (\Delta r_{i+1}) r_{i+1} (d_{23})_{i+1}$$

$$c_{15}{}^i = (\Delta r_i) r_{i+1} (d_{13})_i$$

$$c_{16}{}^i = (\Delta r_i)[r_i (d_{33})_i + r_{i+1}(d_{33})_{i+1}]$$

$$c_{17}{}^i = (\Delta r_i)[r_i (\Delta\sigma_z{}^\circ)_i + r_{i+1}(\Delta\sigma_z{}^\circ)_{i+1}] \tag{32}$$

Now we can form a system of 2n+3 equations for solving 2n+3 unknowns, $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, at $i = 1,2,\ldots,n,n+1$ and $\Delta\varepsilon_z$. Equations (29), (30), and (31) are taken as first and last two equations, respectively, and the other 2n equations are set up at $i = 1,2,\ldots,n$ using equations (25) and (27). The final system is an unsymmetric matrix of arrow type with the nonzero terms appearing in the last row and column and others clustered about the main diagonal, two below and one above.

NUMERICAL RESULTS AND DISCUSSIONS

The numerical results for two particular problems follow. The first problem is a closed-end thick-walled cylinder subjected to varying internal pressure p and temperature T as shown in Figure 1. The heating is uniform throughout the thickness, but the initial yield stress is temperature-dependent as shown in Figure 1. The other material constants are E = 86,666 psi, $\nu = 0.3$, $\omega = 0.0$, $\alpha = 0.0$. The numerical results for the radial displacements (u_a and u_b) at the inside and outside surface are shown in Figure 2. The percentage of plastic zone is also shown in the figure by the dotted line. The entire cylinder is elastic during the time interval 10 to 12. The results for the three stress components at selected time t = 4, 8,

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4. P. C. T. Chen, "Elastic-Plastic Thick-Walled Tubes Subjected to Internal Pressure and Temperature Gradient," Transactions of the Twenty-Seventh Conference of Army Mathematicians, pp. 113-126, January 1982.
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8. K. J. Bathe, "ADINA - A Finite Element Program for Automatic Dynamic Incremental Analysis," Report 82448-1, Acoustics and Vibration Lab., MIT, September 1975, (revised May 1976).

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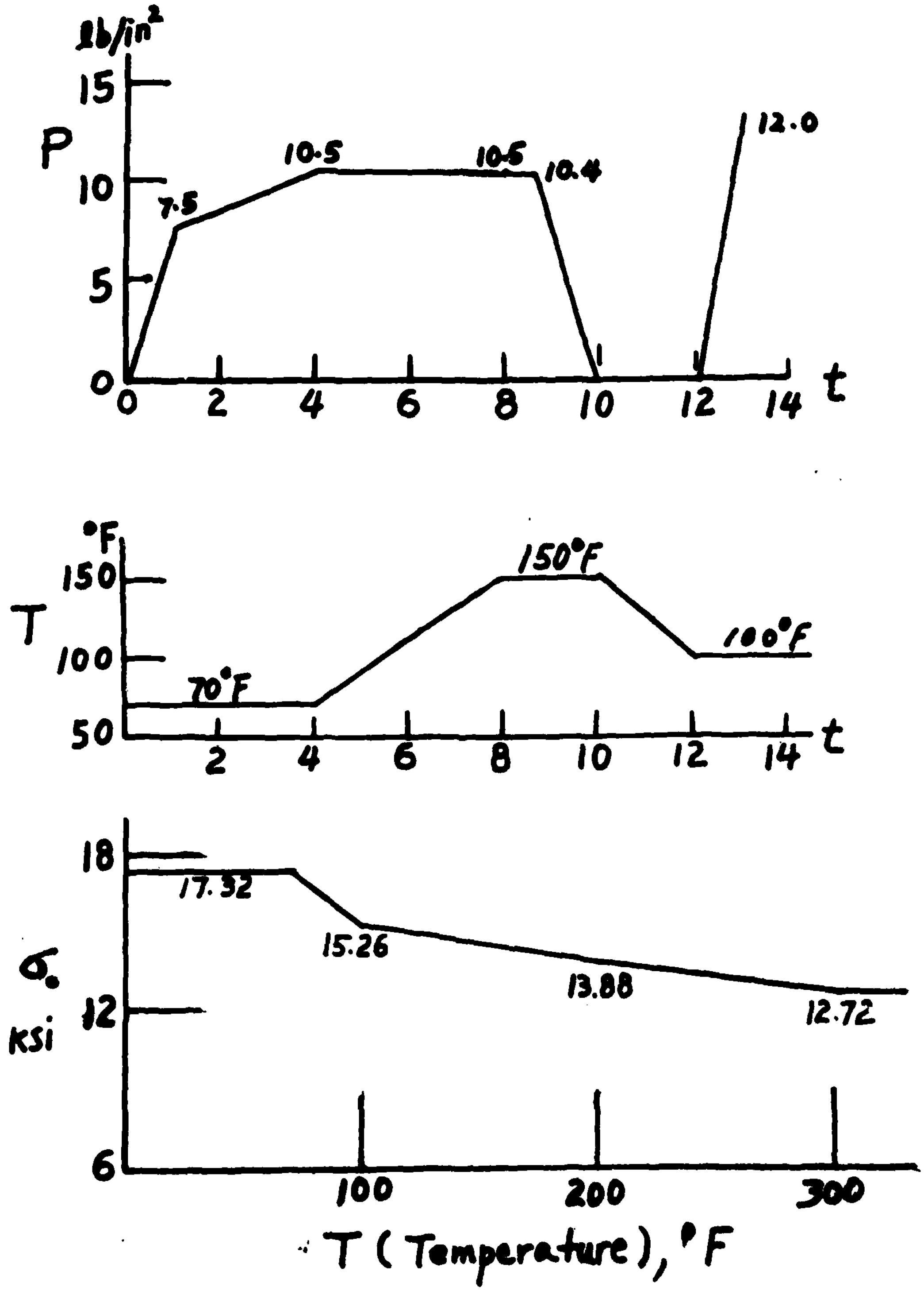

Figure 1. Pressure Temperature History, Temperature-Dependent Yield Stress.

This report presents the numerical results for a closed-end cylinder subjected to varying internal pressure and/or temperature. The thermal problem is due to uniform heating or thermal gradient. If the temperature distribution is solved by a transit analysis, the corresponding thermal stresses can be calculated. The result of this transit thermal problem is to be reported at a later date.

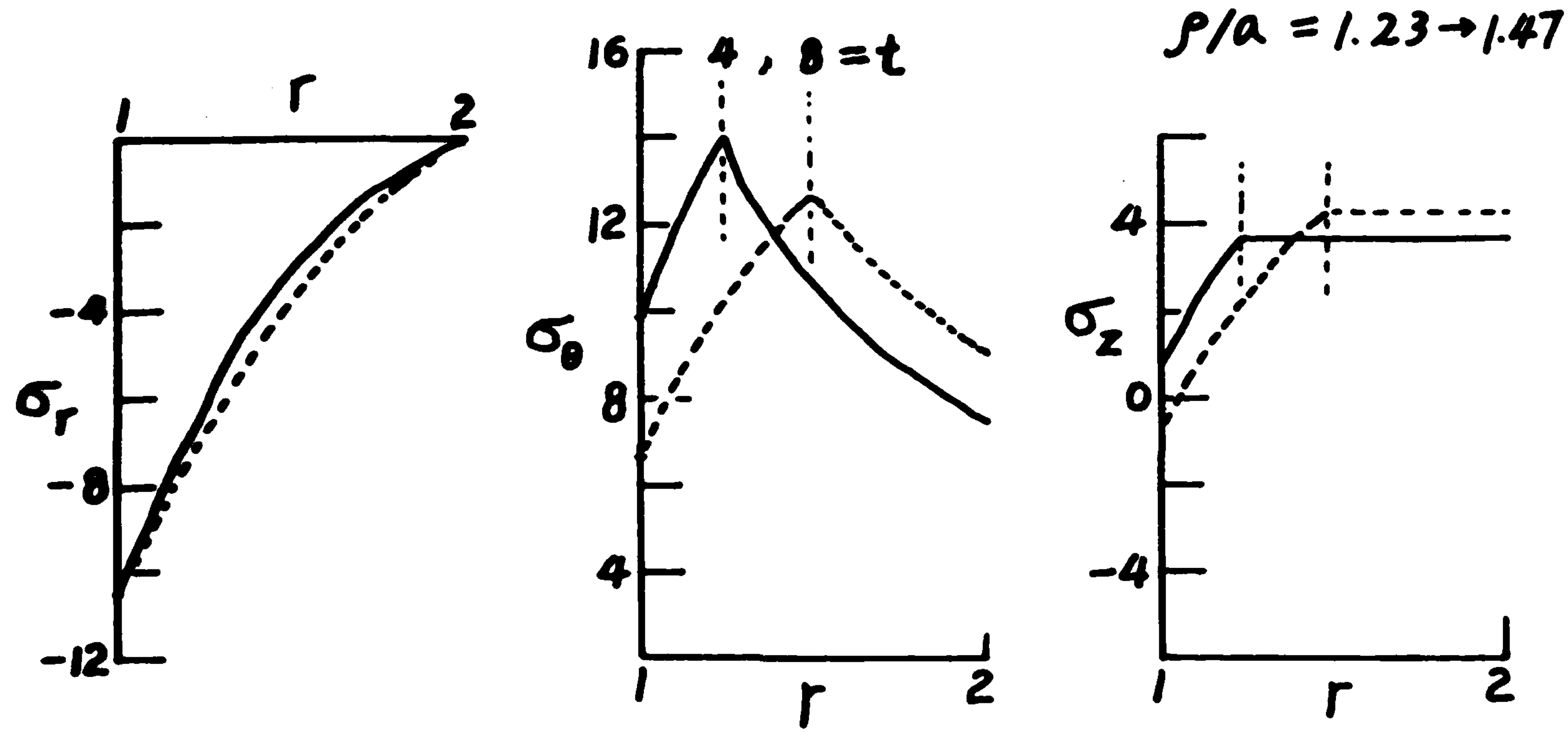

Figure 3. Distribution of Stresses at Time t = 4, 8.

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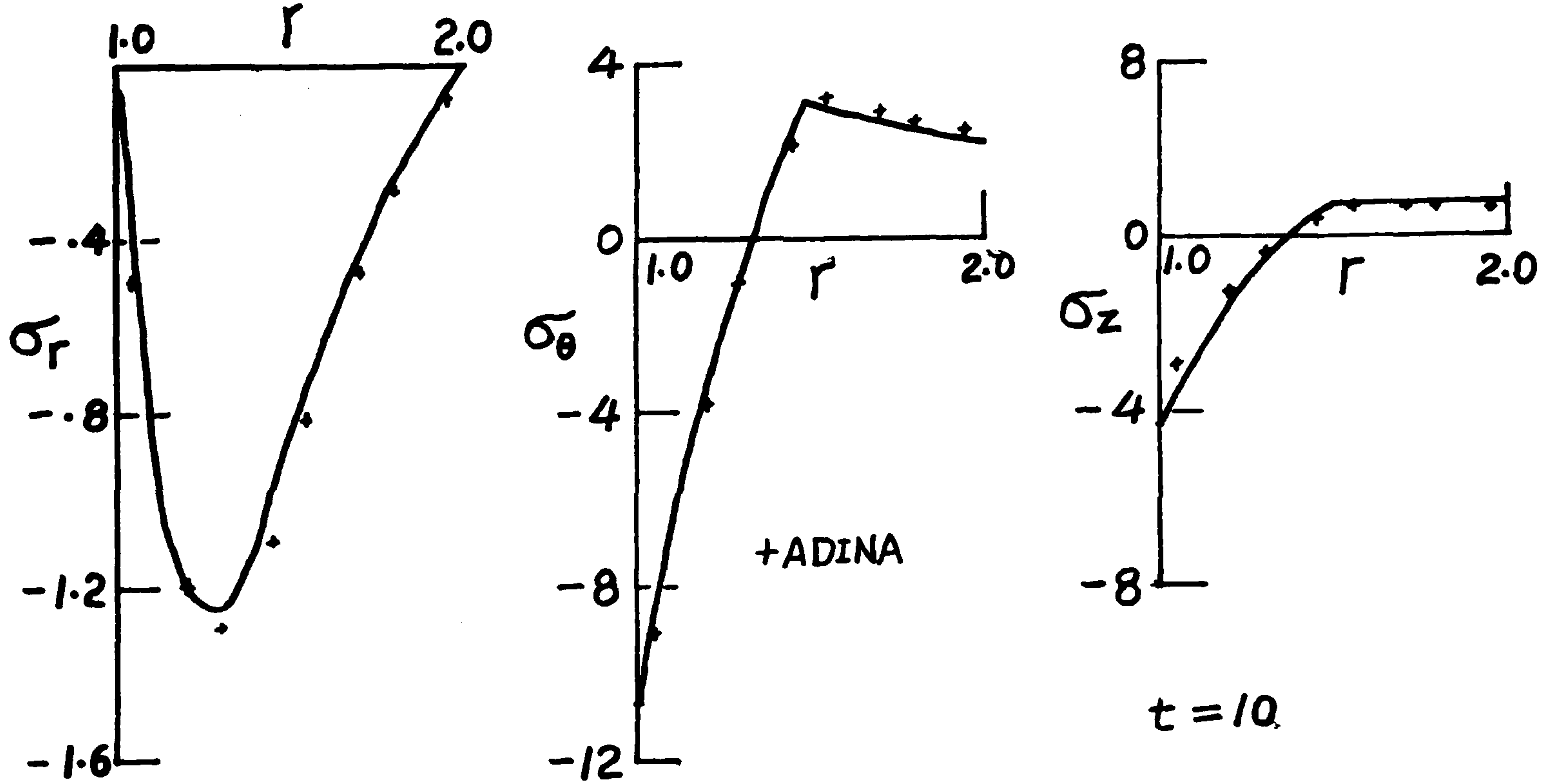


Figure 4. Distribution of (Residual) Stresses at Time t = 10.

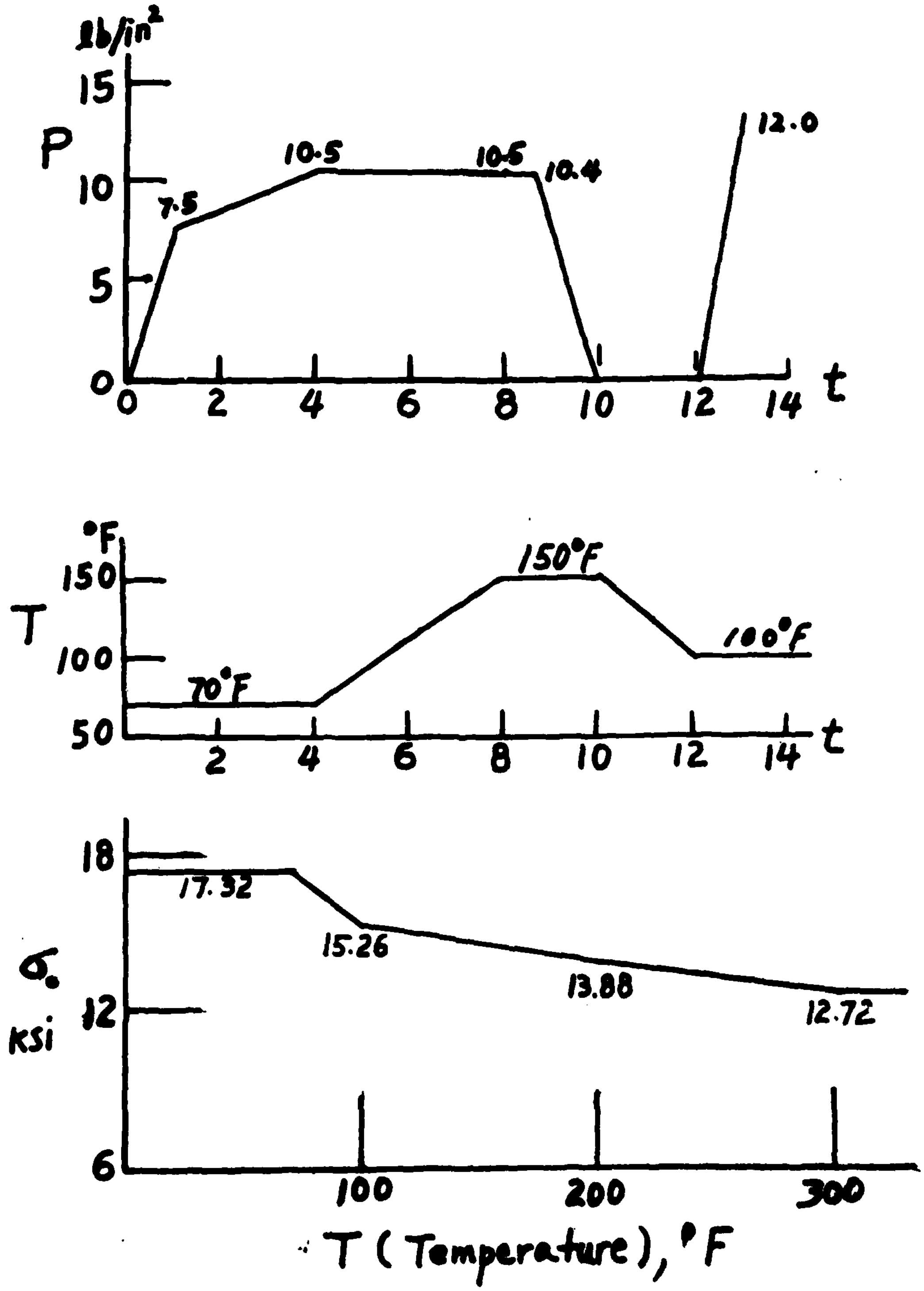


Figure 1. Pressure Temperature History, Temperature-Dependent Yield Stress.

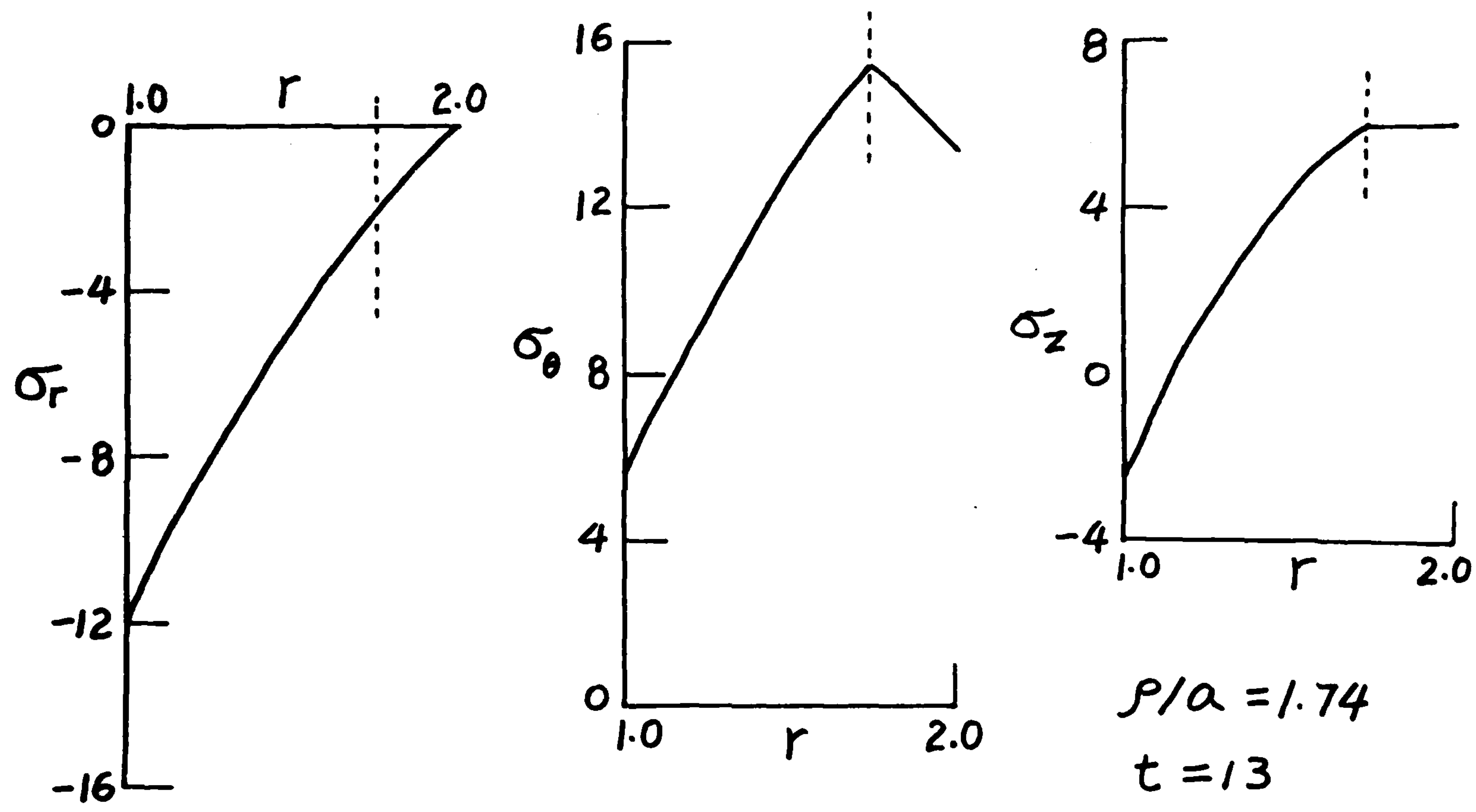
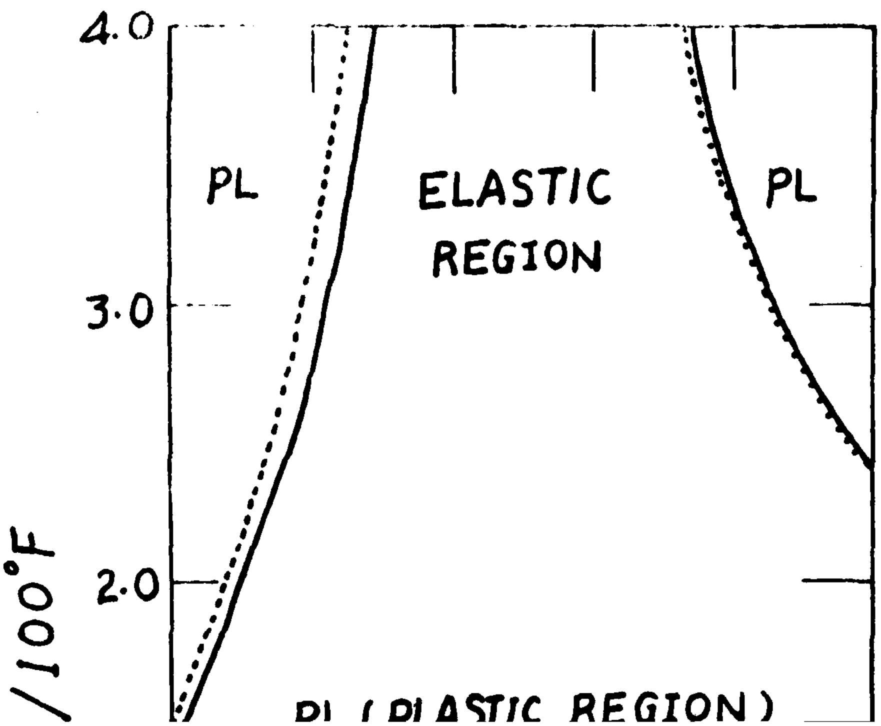


Figure 5. Distribution of Stresses at Time t = 13.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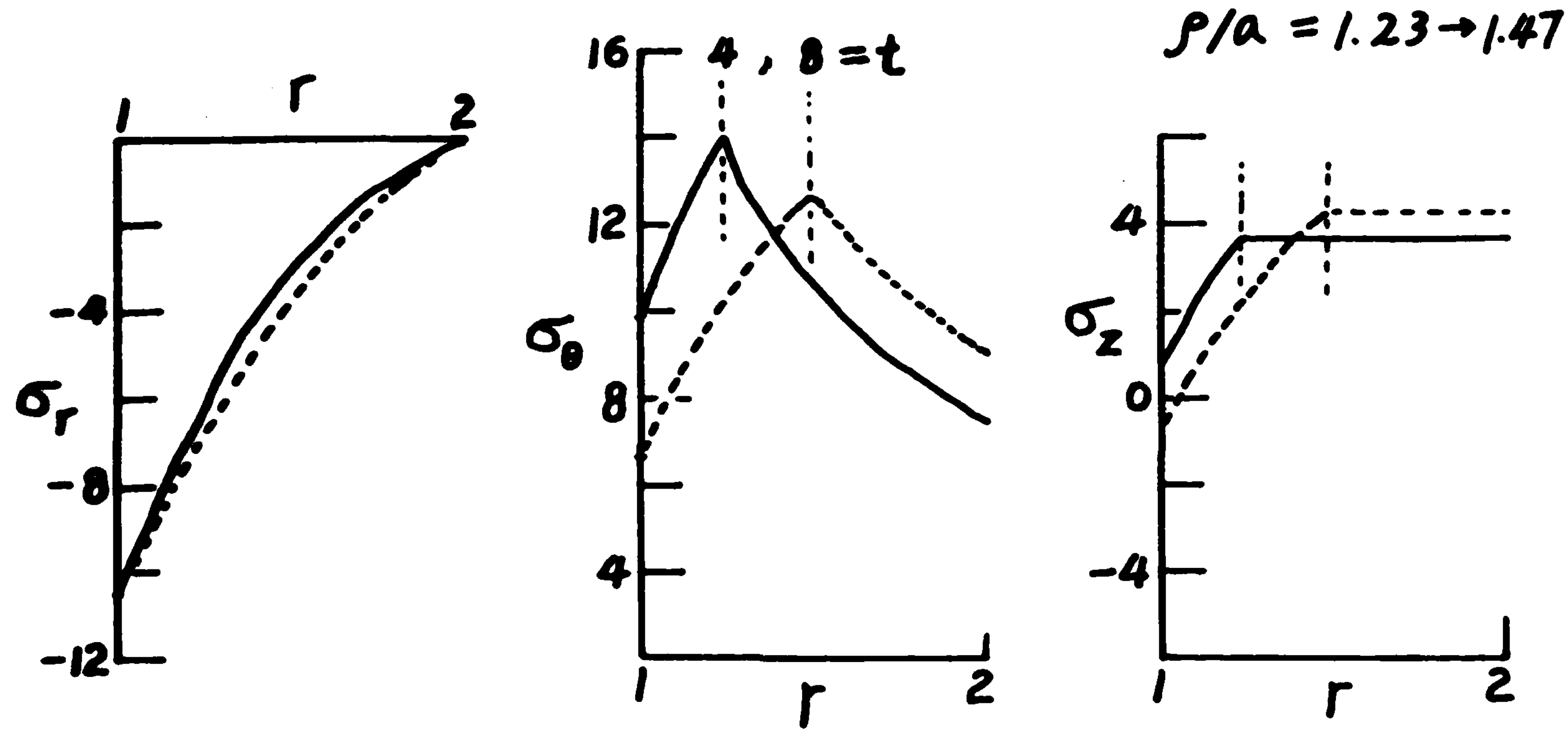

Figure 3. Distribution of Stresses at Time t = 4, 8.

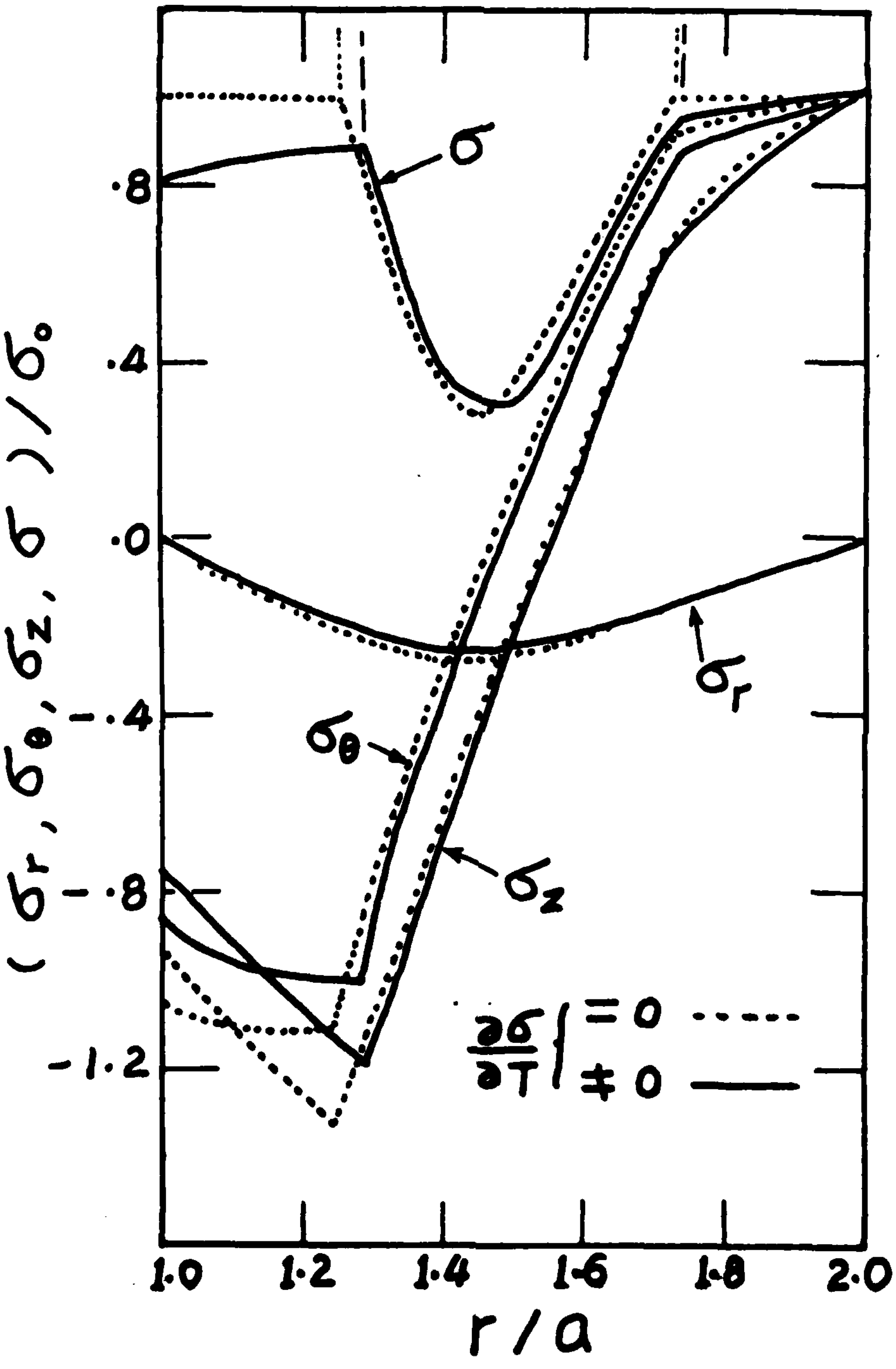

Figure 7. Distribution of Stresses (T_a = 400°F, T_b = 0).

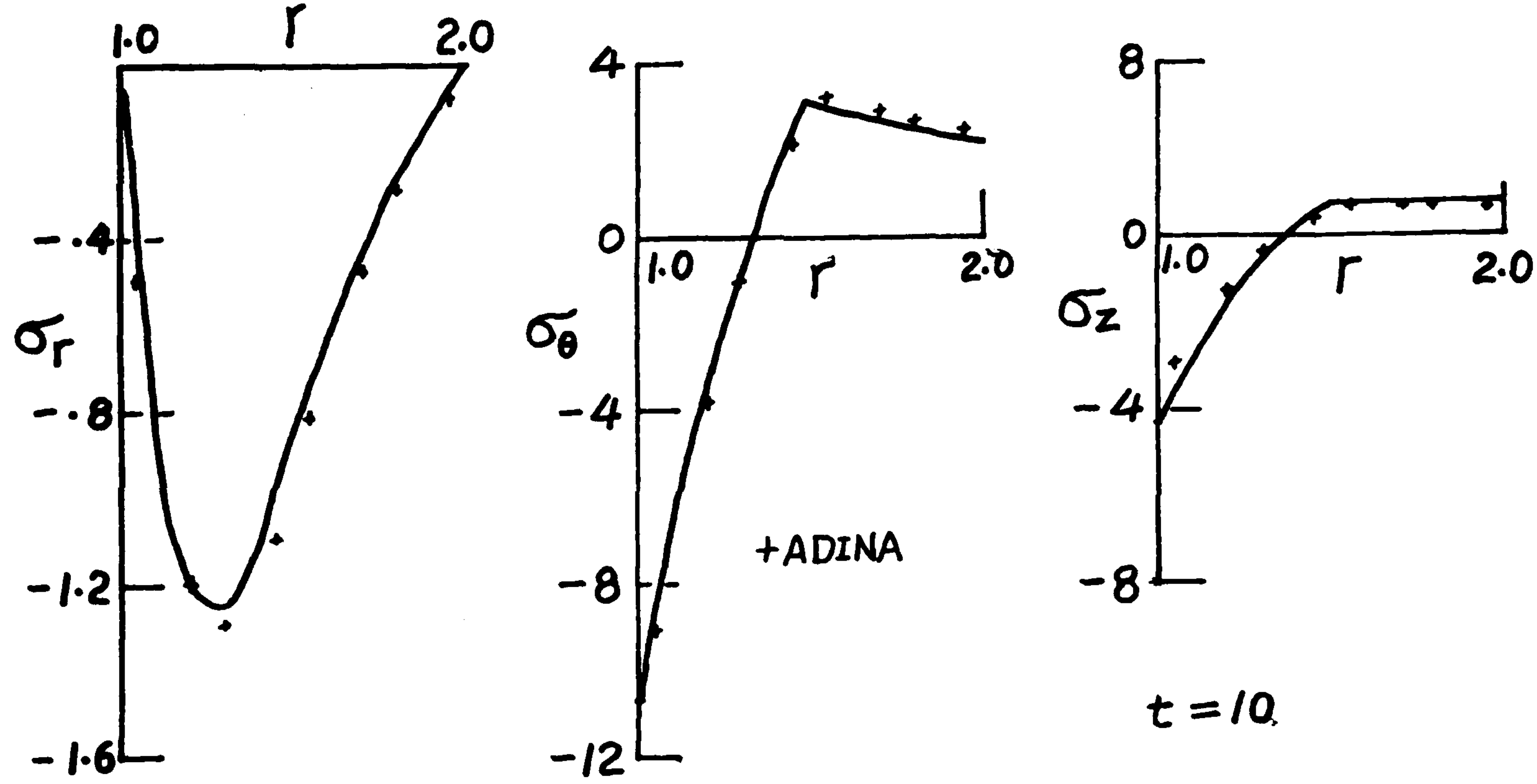

Figure 4. Distribution of (Residual) Stresses at Time t = 10.

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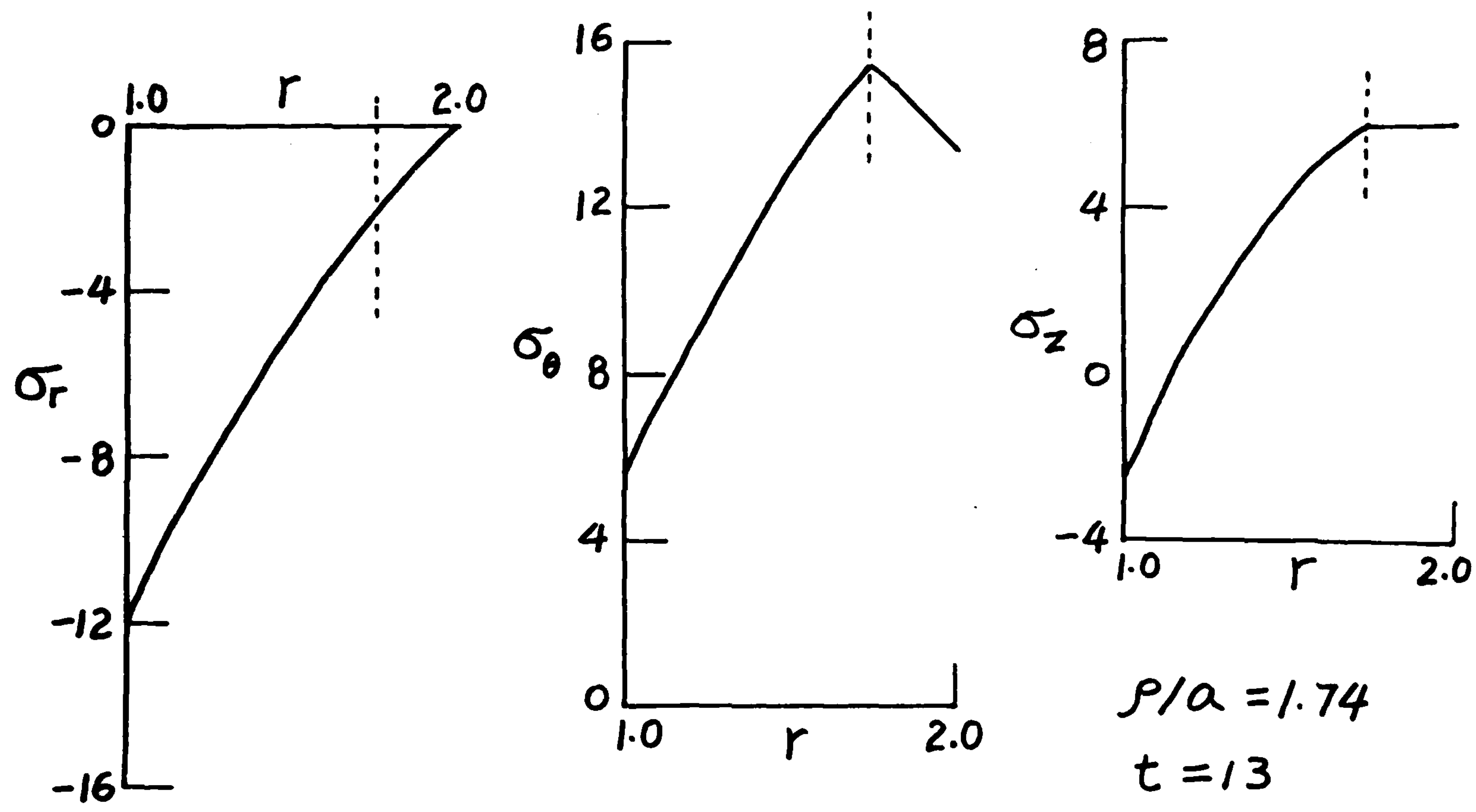

Figure 5. Distribution of Stresses at Time t = 13.

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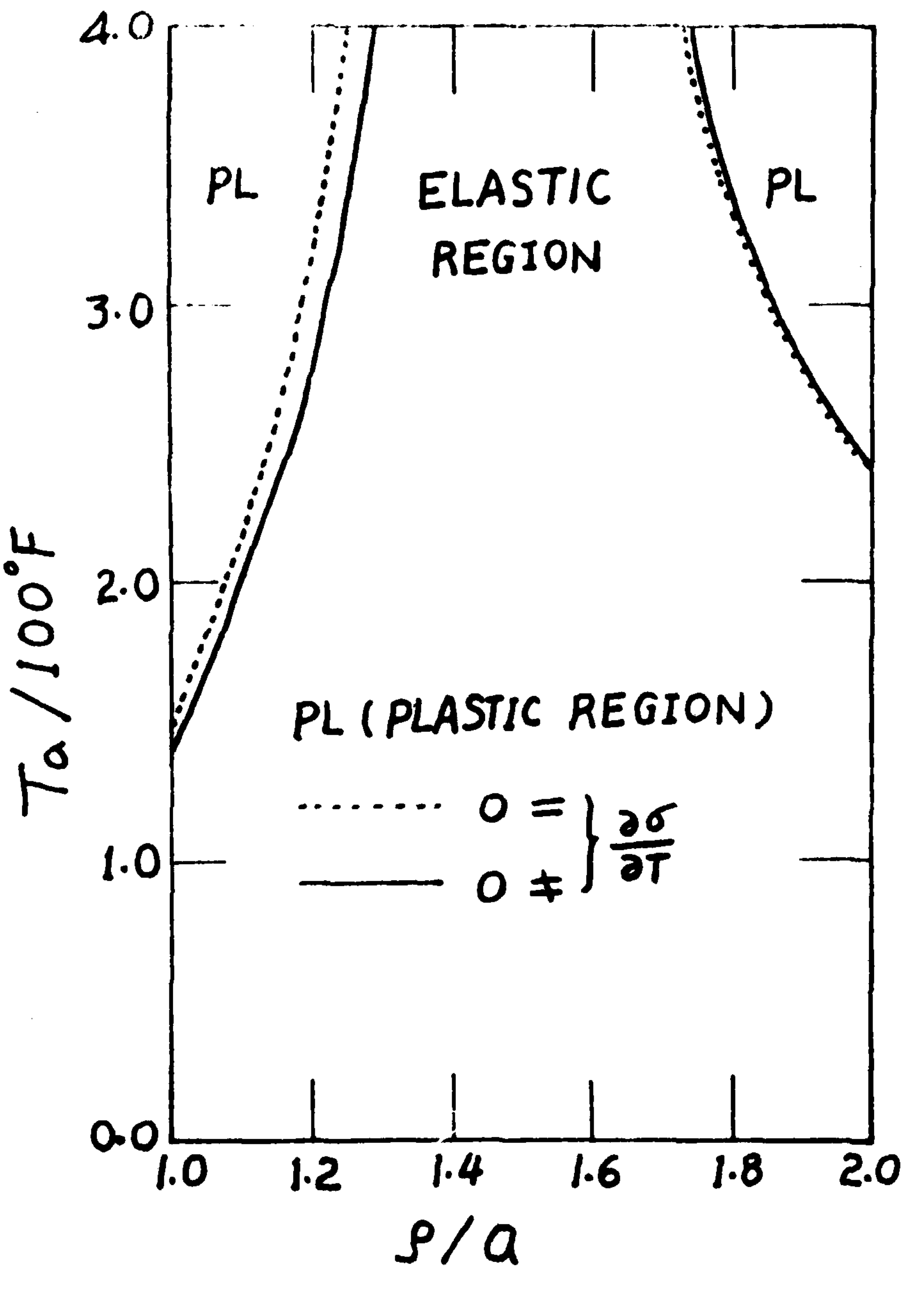

Figure 6. Elastic-Plastic Interface vs. Temperature Gradient.

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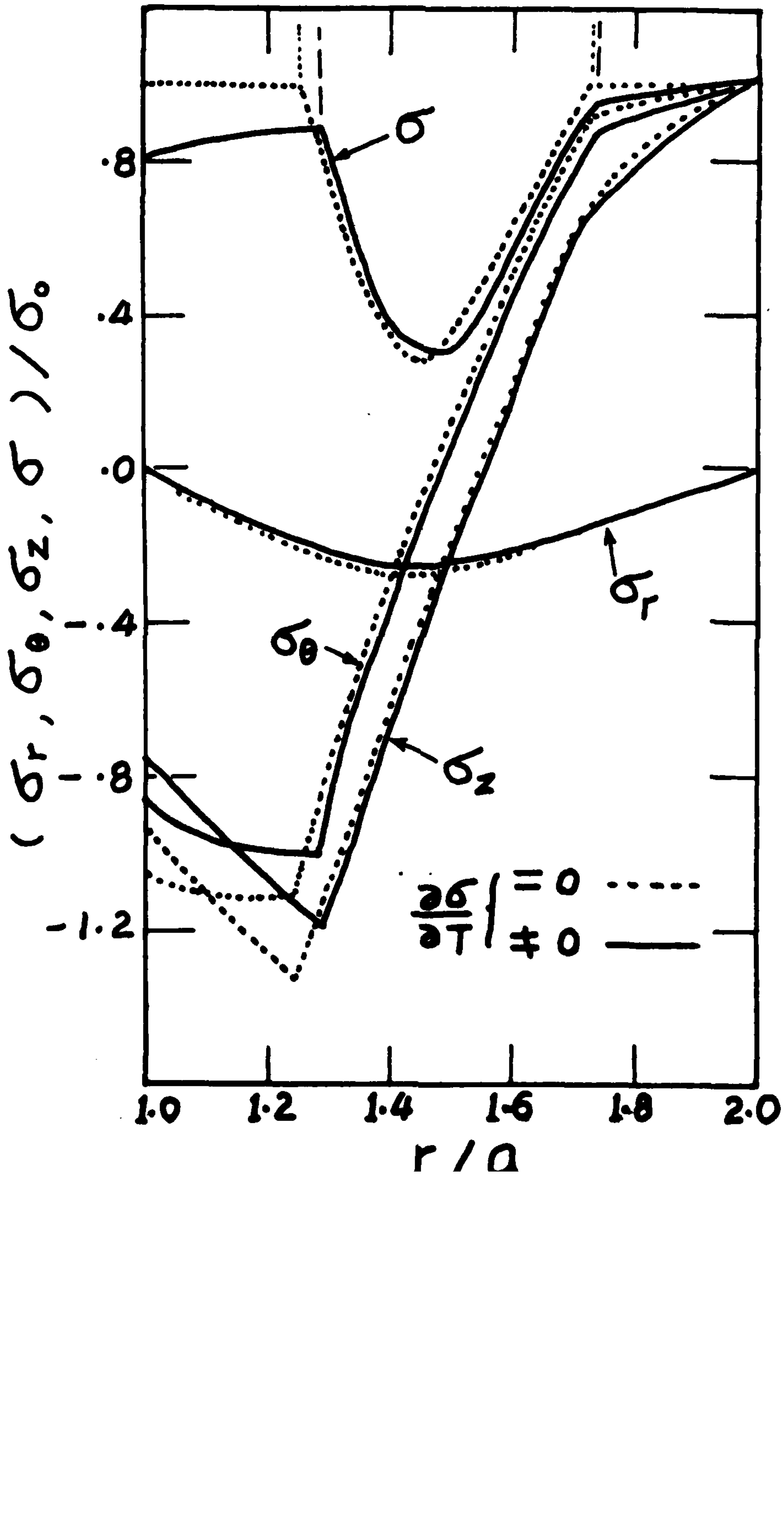

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NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY, DRDAR-LCB-TL, WATERVLIET ARSENAL, WATERVLIET, NY 12189, OF ANY REQUIRED CHANGES.

END

DATE
FILMED

5 – 83

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